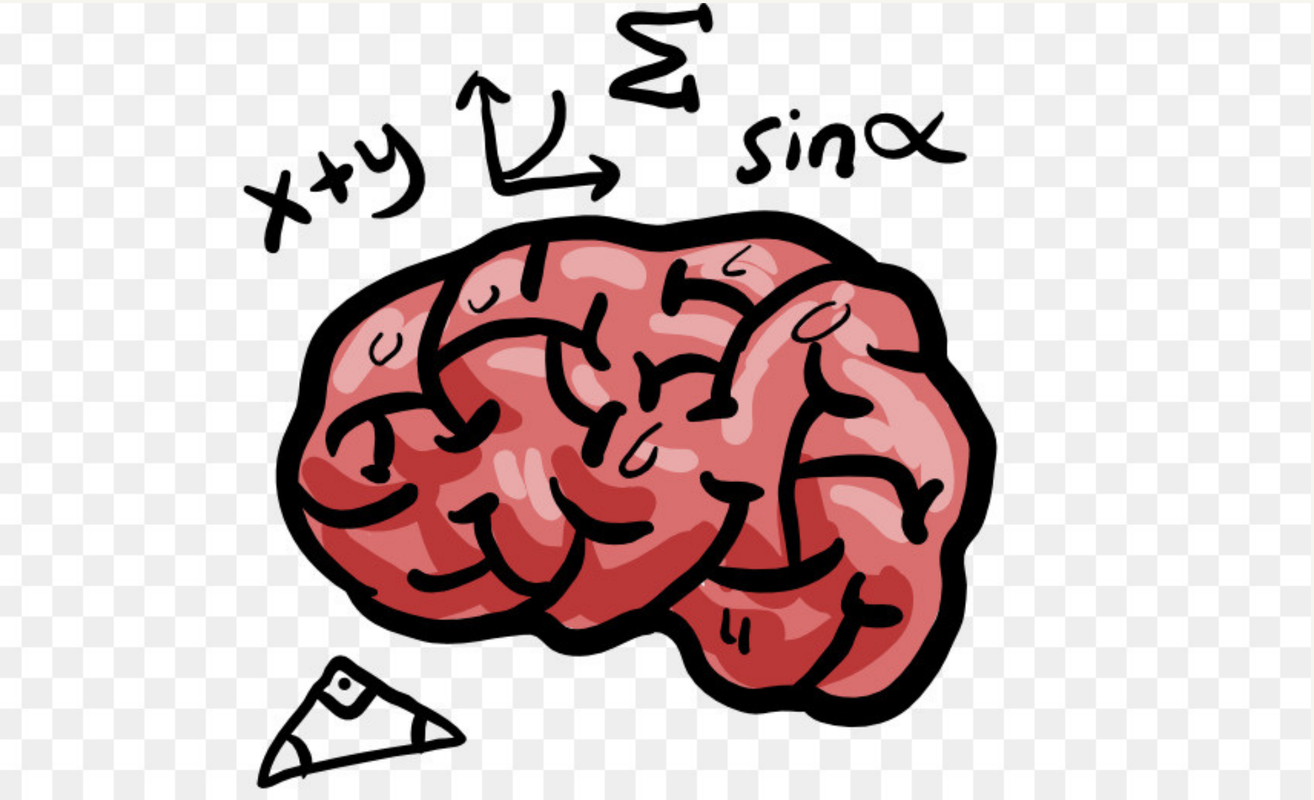
PROBLEME DIN COTIDIAN

realizat de Covalenco Artiom

CONDIȚIA PROBLEMEI

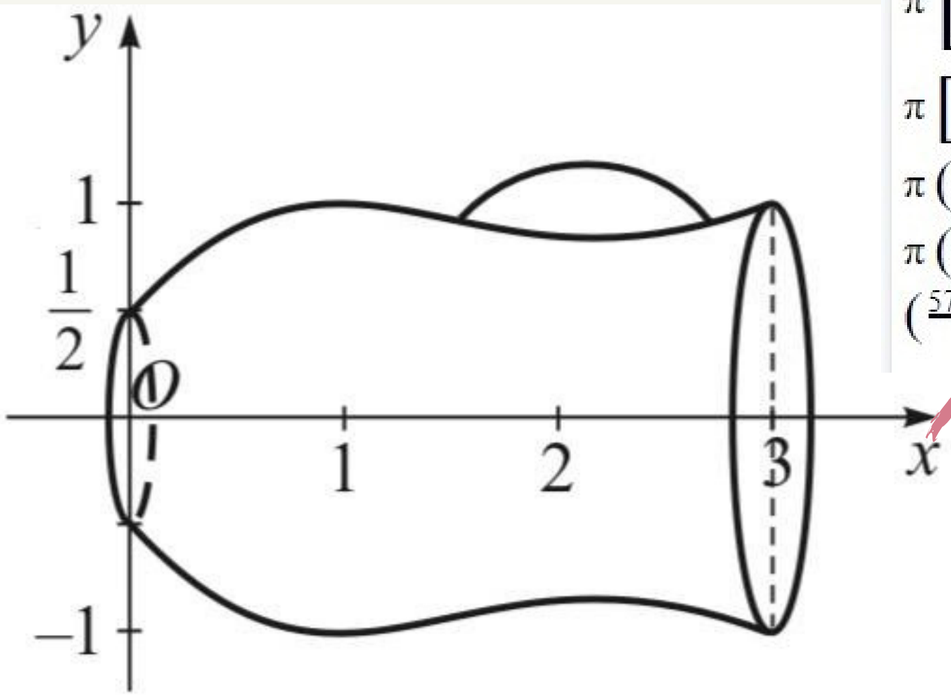
Determinaţi volumul unui urcior care se obţine prin rotirea în jurul axei Ox a subgraficului funcţiei



SE DĂ

$f:[0,3]\to\mathbb{R}$,

$$f(x)=\begin{cases}-\frac{x^2}{2}+x+\frac{1}{2}, & \text{dacă } 0\le x\le 2\\ \frac{3}{2}x^2-7x+\frac{17}{2}, & \text{dacă } 2\le x\le 3.\end{cases}$$



REZOLVAREA

$$V(C_f)=\pi\int_0^3 f^2(x)dx=\pi\left[\int_0^2\left(-\frac{x^2}{2}+x+\frac{1}{2}\right)dx+\int_2^3\left(\frac{3}{2}x^2-7x+\frac{17}{2}\right)^2dx\right]=$$

$$\pi\left[\int_0^2\left(\frac{x^4}{4}+x^2+\frac{1}{4}-x^3-\frac{1}{2}x^2+x\right)dx+\int_2^3\left(\frac{9}{4}x^4+49x^2+\frac{289}{4}-21x^3+\frac{51}{2}x^2-119x\right)dx\right]=$$

$$\pi\left[\int_0^2\left(\frac{x^4}{4}+\frac{1}{2}x^2-x^3+x+\frac{1}{4}\right)dx+\int_2^3\left(\frac{9}{4}x^4+\frac{149}{2}x^2-21x^3-119x+\frac{289}{4}\right)dx\right]=$$

$$\pi\left[\left(\frac{x^5}{20}+\frac{x^3}{6}-\frac{x^4}{4}+\frac{x^2}{2}+\frac{1}{4}\right)\Big|_0^2+\left(\frac{9x^5}{20}+\frac{149x^3}{6}-\frac{21x^4}{4}-\frac{119x^2}{2}+\frac{289x}{4}\right)\Big|_2^3\right]=$$

$$\pi\left(\frac{32}{20}+\frac{8}{6}-\frac{16}{4}+\frac{4}{2}+\frac{2}{4}+\frac{2187}{20}+\frac{1341}{2}-\frac{1701}{4}-\frac{1701}{2}+\frac{876}{4}-\frac{288}{20}-\frac{596}{3}+84+238-\frac{289}{2}\right)=$$

$$\pi\left(\frac{1931}{20}-\frac{592}{3}+82+238+\frac{271}{2}-\frac{289}{2}-\frac{417}{2}\right)=\left(\frac{1931}{20}-\frac{592}{3}+311-\frac{417}{2}\right)\pi=$$

$$\left(\frac{5793+18660-11840-12510}{60}\right)\pi=\frac{103}{60}\pi(u.c)$$