Problema :
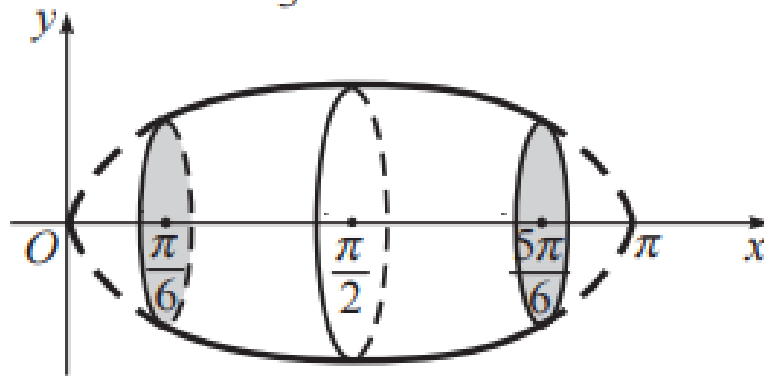
Să se afle volumul unui butoi, știind că doagele lui au forma unui arc de sinusoidă $y = a \sin x$ $(a > 0)$, iar lungimea butoiului este $\frac{2\pi}{3}$.

Datele :

Y = a sinx (a>0)

L = 2π/3

Rezolvare :

V (Cf) = $\pi \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (a\sin x)^2 dx =$

$$\pi \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(a^2 \sin^2 x\right) dx =$$

$$\pi \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} a^2 \left(\frac{1 - \cos^2 x}{2}\right) dx =$$

$$\pi \frac{a^2}{2} \left(x - \frac{1}{2}\sin x\right) \Big|_{\frac{\pi}{6}}^{\frac{5\pi}{6}} =$$

$$\pi \frac{a^2}{2} \left(\frac{5\pi}{6} - \frac{1}{2} sin \frac{5\pi}{3} - \frac{\pi}{6} + \frac{1}{2} sin \frac{\pi}{3} \right) =$$

$$\pi \frac{a^2}{2} \left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2} \right) =$$

$$\pi \frac{a^2}{2} \left(\frac{4\pi + 3\sqrt{3}}{6} \right) =$$

$$\pi a^2 \left(\frac{4\pi + 3\sqrt{3}}{12} \right) (u.c.)$$